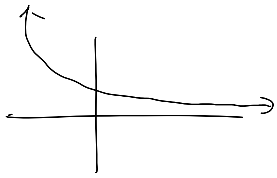
Given the exponential function: $g(x)=\left(\frac{1}{2}\right)^x = 2^{-x}$



Range $y>0$

$y \in (0, \infty)$

$0 < y < \infty$

4.1 Write down the range of g.

4.2 Determine the equation of g^{-1} in the form $y = \ldots$

4.3 Is g^{-1} a function? Justify your answer.

4.4 The point M(a ; 2) lies on g^{-1}.

4.4.1 Calculate the value of a.

4.4.2 M′, the image of M, lies on g. Write down the coordinates of M′.

4.5 If $h(x) = g(x+3)+2$, write down the coordinates of the image of M′ on h.

shift up by 2

shift to the left by 3

$M\left(\frac{1}{4}, 2\right)$

$M'\left(2, \frac{1}{4}\right)$

4.2 $y=\left(\frac{1}{2}\right)^x$

$x=\left(\frac{1}{2}\right)^y \qquad x \leftrightarrow y$

we can now solve for y

$x = 2^{-y}$

$\log_2(x) = \log_2(2^{-y})$

$\log_2(x) = -y$

$y = -\log_2(x) \qquad = g^{-1}$

Yes, g^{-1} is a function because g^{-1} takes one input and sends it to one output!

4.4.1) $M(a, 2)$ lies on g^{-1}

$y = -\log_2(x)$

$2 = -\log_2(a)$

$-2 = \log_2(a)$

$a = \frac{1}{4}$

$g^{-1}(a) = 2$

$g(g^{-1}(a)) = g(2)$

$a = \frac{1}{4}$

4.5) $M'\left(2, \frac{1}{4}\right)$

Shift left by 3 and up by 2

$M''\left(2-3, \frac{1}{4}+2\right)$

$M''\left(-1, \frac{9}{4}\right)$

Pg2

Monday, 09 May 2022 12:18

5.1 Given: $f(x)=\frac{1}{x+2}+3$ ← shifted hyperbola

5.1.1 Determine the equations of the asymptotes of f.

5.1.2 Write down the y-intercept of f.

5.1.3 Calculate the x-intercept of f.

5.1.4 Sketch the graph of f. Clearly label ALL intercepts with the axes and any asymptotes.

5.1.1) Horizontal Asymptote : $y=3$

Vertical Asymptote : $x=-2$

5.1.2) y-intercept $x=0$

$$y=\frac{1}{0+2}+3=\frac{1}{2}+3=\frac{7}{2} \quad \left(0;\frac{7}{2}\right)$$

5.1.3) x-intercept $y=0$

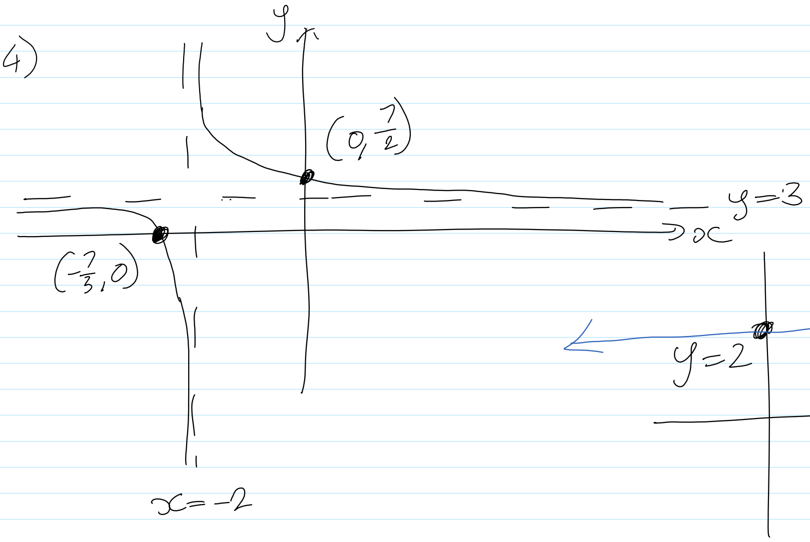
$$0=\frac{1}{x+2}+3$$

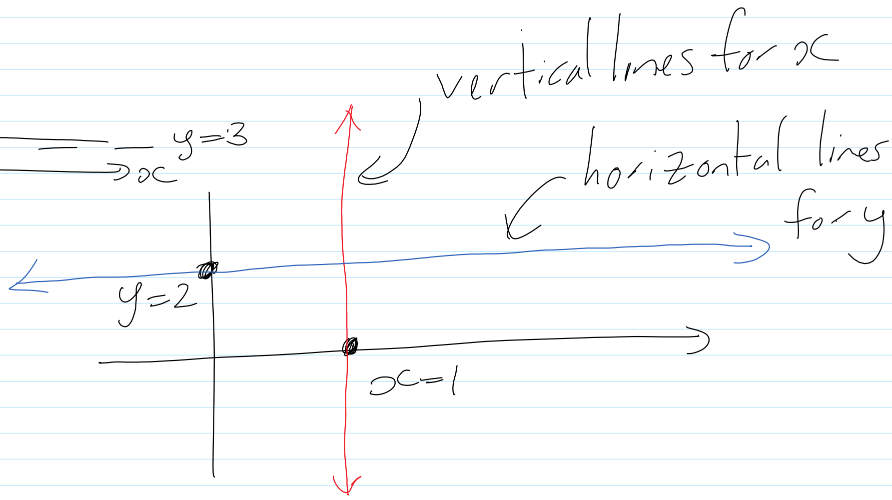
$$-3=\frac{1}{x+2}$$

$$-\frac{1}{3}=x+2$$

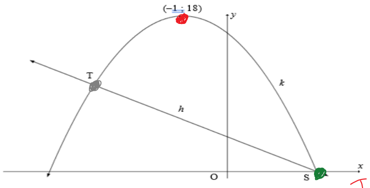
$$x=-\frac{7}{3} \quad \left(-\frac{7}{3},0\right)$$

5.1.4)





5.2 Sketched below are the graphs of $k(x) = ax^2 + bx + c$ and $h(x) = -2x + 4$. Graph k has a turning point at (–1 ; 18). S is the x-intercept of h and k. Graphs h and k also intersect at T.



5.2.1 Calculate the coordinates of S.

5.2.2 Determine the equation of k in the form $y = a(x+p)^2 + q$ ← T.P form

5.2.3 If $k(x) = -2x^2 - 4x + 16$, determine the coordinates of T.

5.2.4 Determine the value(s) of x for which $k(x) < h(x)$.

5.2.5 It is further given that k is the graph of $g'(x)$ ← gradient of g

(a) For which values of x will the graph of g be concave up?

(b) Sketch the graph of g, showing clearly the x-values of the turning points and the point of inflection.

5.2.1)
$h(x) = -2x + 4$
$0 = -2x + 4$
$-4 = -2x$
$x = 2$
$S(2,0)$

5.2.2)
$k(x) = a(x+p)^2 + q$
$k(x) = a(x+1)^2 + 18$
$0 = a(2+1)^2 + 18$
$0 = a \cdot 9 + 18$
$-18 = 9a$
$a = -2$

$$k(x) = -2(x+1)^2 + 18$$

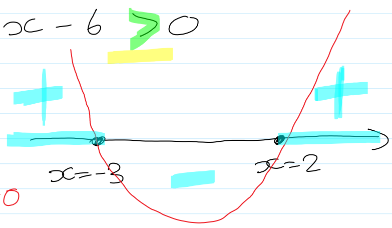
5.2.3) $k(x) = h(x)$
$-2x^2 - 4x + 16 = -2x + 4$
$-2x^2 - 2x + 12 = 0$
$x^2 + x - 6 = 0$
$(x+3)(x-2) = 0$

$x + 3 = 0$ → $x = -3$ corresponds to T
$x - 2 = 0$ → ~~$x = 2$~~ corresponds to S

$x = -3 \Rightarrow y = -2(-3) + 4 = 10$
$T(-3, 10)$

5.2.4) $k(x) < h(x)$
$-2x^2 - 4x + 16 < -2x + 4$
$-2x^2 - 4x + 2x + 16 - 4 < 0$
$x^2 + x - 6 > 0$

$x^2 + x - 6 = 0$
$x = -3, x = 2$
$(x+3)(x-2) = 0$



We are interested in $x < -3$ OR $x > 2$

$x \in (-\infty, -3) \cup (2, \infty)$

We are interested in $x < -3$ OR $x > 2$

$x \in (-\infty, -3) \cup (2, \infty)$

5.2.5) $k(x) = g'(x)$

$g''(x) = k'(x) = -4x - 4$

Concave Up $\Longleftrightarrow$ $g''(x) > 0$, Concave Down $g''(x) < 0$

$-4x - 4 > 0$

$4x + 4 < 0$

$\boxed{x < -1}$

Concave up on the interval $(-\infty, -1)$

b) $g'(x) = 0$

$-2x^2 - 4x + 16 = 0$

$(x+4)(x-2) = 0$

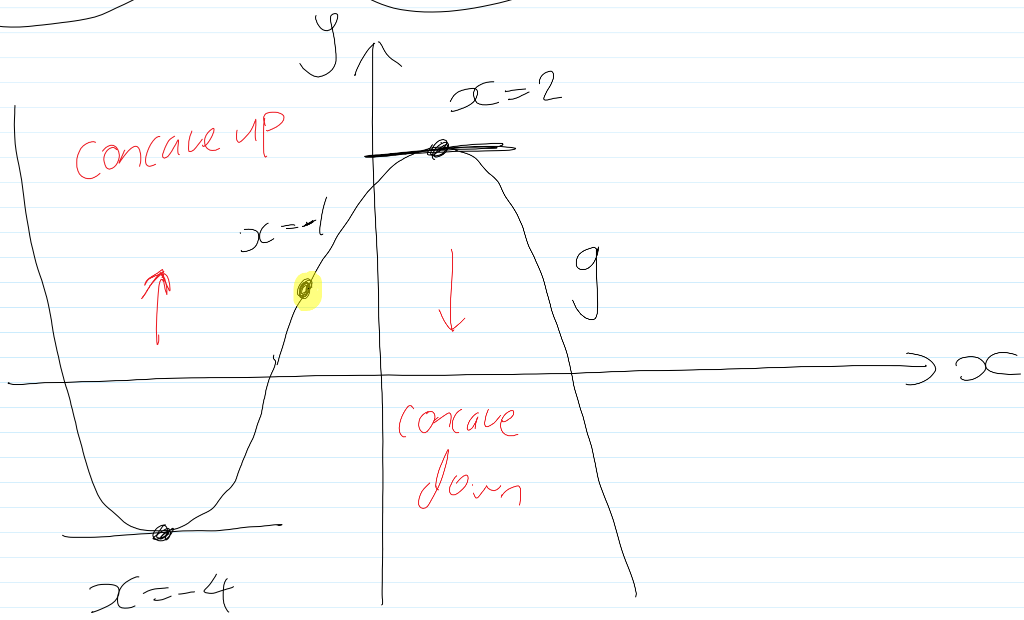
$x + 4 = 0$ → $x = -4$

$x - 2 = 0$ → $x = 2$

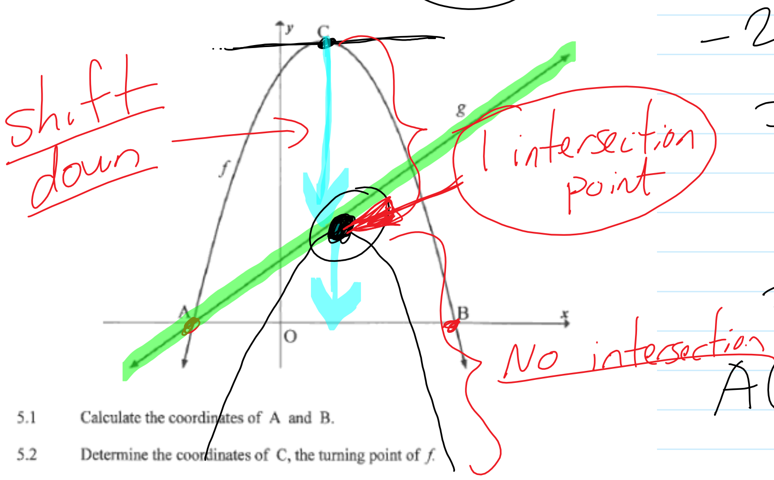
$g''(x) = 0$

$-4x - 4 = 0$

$x = -1$



Sketched below are the graphs of $f(x) = -2x^2 + 4x + 16$ and $g(x) = 2x + 4$.
A and B are the x-intercepts of f. C is the turning point of f.



5.1 Calculate the coordinates of A and B.

5.2 Determine the coordinates of C, the turning point of f.

5.3 Write down the range of f.

5.4 The graph of $h(x) = f(x + p) + q$ has a maximum value of 15 at $x = 2$. Determine the values of p and q.

5.5 Determine the equation of g^{-1}, the inverse of g, in the form $y = \ldots$

5.6 For which value(s) of x will $g^{-1}(x).g(x) = 0$?

5.7 If $p(x) = f(x) + k$, determine the value(s) of k for which p and g will NOT intersect.

taking the parabola and shifting up/down

$$-2x^2 + 4x + 16 = 0$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x = 4 \text{ OR } x = -2$$
$$A(-2,0) \quad B(4,0)$$

5.2) $f'(x) = 0$

$$\frac{d}{dx}\left(-2x^2 + 4x + 16\right) = 0$$
$$-4x + 4 = 0$$
$$4x = 4$$
$$x = 1 \Rightarrow f(1) = -2(1)^2 + 4(1) + 16$$
$$= 18$$

$C(1, 18)$

OR we could complete the square

$$f(x) = -2x^2 + 4x + 16$$
$$= -2\left[x^2 - 2x\right] + 16$$
$$= -2\left[(x-1)^2 - 1\right] + 16$$
$$= -2(x-1)^2 + 2 + 16$$
$$= -2(x-1)^2 + 18$$

$= -2(x-1)^2 + 2 + 16$

$= -2(x-1)^2 + 18$

$C(1, 18)$

5.3) sad face parabola so we include everything below the turning point

$y \leq 18$, $(-\infty, 18]$

5.4) $h(x) = f(x+p) + q$ TP(2, 15)

$h(x) = -2(x-1+p) + 18 + q$

$p - 1 = -2$ $18 + q = 15$

$p = -1$ $q = -3$

5.5) $g(x) = 2x + 4$

$y = 2x + 4$

$x = 2y + 4$, $x \leftrightarrow y$

* we solve for y which is g^{-1}

$x - 4 = 2y$

$y = \frac{1}{2}x - 2$ $g^{-1}(x)$

5.6) $g^{-1}(x) \cdot g(x) = 0$

$(\frac{1}{2}x - 2)(2x + 4) = 0$ * already factored

$(\frac{1}{2}x-2)(2x+4)$ factored

$\frac{1}{2}x-2=0$ OR $2x+4=0$

$\frac{1}{2}x=2$ $\quad$ $2x=-4$

$x=4$ $\quad$ $x=-2$

We have TWO Solutions ($x=4$, $x=-2$) for which the product is 0.

5.7) $p(x)=g(x)$ "shifted parabola = line"

$-2x^2+4x+16+k=2x+4$

$-2x^2+2x+(12+k)=0$ the roots give us the intersection points

$\Delta = b^2-4ac=0$ ← $\Delta=0$ ⇓ one intersection point

$(2)^2-4(-2)(12+k)=0$

$4+8(12+k)=0$

$1+2(12+k)=0$

$1+24+2k=0$

$2k=-25$

$k=-\frac{25}{2}$ (one intersection)

$k<-\frac{25}{2}$ (NOT EQUAL)

A larger shift down (more negative)

No intersection anymore

No intersection anymore

Pg5

Monday, 09 May 2022 12:21

Given: $g(x) = 3^x$

6.1.1 Write down the equation of g^{-1} in the form $y = \dots$

6.1.2 Point P(6 ; 11) lies on $h(x) = 3^{x-4} + 2$. The graph of h is translated to form g. Write down the coordinates of the image of P on g.

$h(x)$ is $g(x)$ but translated
- 4 units to the right
- 2 units up

6.1.) $g(x) = 3^x$

$y = 3^x$

$x = 3^y \quad x \leftrightarrow y$

$\log_3(x) = \log_3(3^y)$

$\log_3(x) = y \cdot \log_3(3)$

$y = \log_3(x)$ (this is our g^{-1})

6.1.2) P(6,11) lies on h

find P' which is the image on g

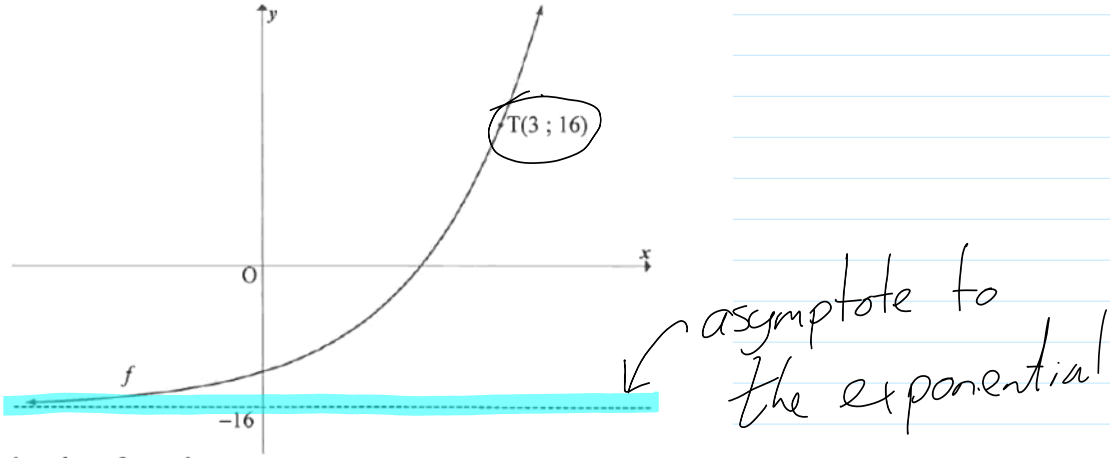
$P'(6-4, 11-2)$

$= P'(2, 9)$

Pg6

Monday, 09 May 2022 12:22

Sketched is the graph of $f(x) = 2^{x+p} + q$. T(3 ; 16) is a point on f and the asymptote of f is $y = -16$.



Determine the values of p and q.

$q = -16$ →

$f(x) = 2^{x+p} - 16$, we can substitute the point T

$16 = 2^{3+p} - 16$

$32 = 2^{3+p}$

$2^5 = 2^{3+p}$

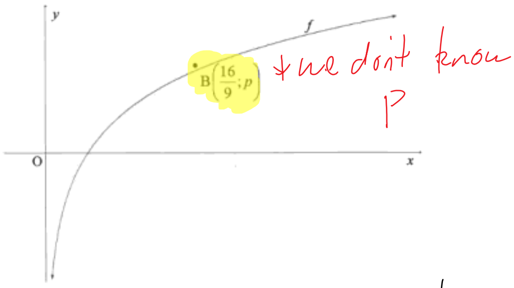
$5 = 3 + p$

$p = 2$ →

Pg7

Thursday, 12 May 2022 06:58

The graph of $f(x) = \log_{\frac{4}{3}} x$ is drawn below. $B\left(\frac{16}{9}; p\right)$ is a point on f.



4.1 For which value(s) of x is $\log_{\frac{4}{3}} x \leq 0$? $0 < x \leq 1$

4.2 Determine the value of p, without the use of a calculator.

4.3 Write down the equation of the inverse of f in the form $y = \ldots$

4.4 Write down the range of $y = f^{-1}(x)$.

$y = \log_{\frac{4}{3}} x$

$p = \log_{\frac{4}{3}}\left(\frac{16}{9}\right)$

$p = \log_{\frac{4}{3}}\left(\left[\frac{4}{3}\right]^2\right)$

$p = 2\log_{\frac{4}{3}}\left(\frac{4}{3}\right)$

$\boxed{p = 2}$

4.3) $y = \log_{\frac{4}{3}} x$

$x = \log_{\frac{4}{3}} y$, $x \leftrightarrow y$

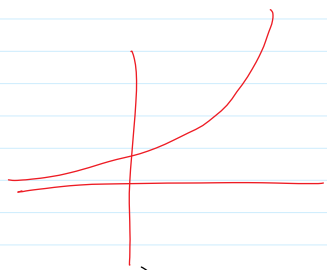
We need to solve for y !

$\left(\frac{4}{3}\right)^x = \left(\frac{4}{3}\right)^{\log_{\frac{4}{3}}(y)}$

$\left(\frac{4}{3}\right)^x = y$ (this is our f^{-1})

4.4) Range of f^{-1}

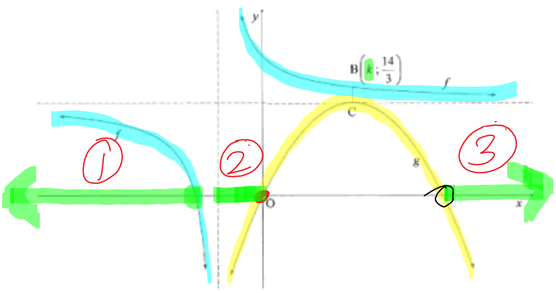
Notice that f^{-1} is an exponential



$y > 0$, $(0, \infty)$, $y \in (0, \infty)$

The graphs of $f(x)=\frac{2}{x+1}+4$ and parabola g are drawn below.

- C, the turning point of g, lies on the horizontal asymptote of f.
- The graph of g passes through the origin.
- B$\left(k;\frac{14}{3}\right)$ is a point on f such that BC is parallel to the y-axis.



5.1 Write down the domain of f.

5.2 Determine the x-intercept of f.

5.3 Calculate the value of k.

5.4 Write down the coordinates of C.

5.5 Determine the equation of g in the form $y=a(x+p)^2+q$. TP

5.6 For which value(s) of x will $\frac{f(x)}{g(x)}\leq 0$?

5.7 Use the graphs of f and g to determine the number of real roots of $\frac{2}{x}-5=-(x-3)^2-5$. Give reasons for your answer.

5.1) $f(x)=\frac{2}{x+1}+4$

$x+1=0$

$x=-1$ VA

Domain: $x\in\mathbb{R}$, $x\neq -1$

$(-\infty,-1)\cup(-1,\infty)$

5.2) x-intercept $(y=0)$

$0=\frac{2}{x+1}+4$

$-4=\frac{2}{x+1}$

$-2=\frac{1}{x+1}$

$-\frac{1}{2}=x+1$

$x=-\frac{3}{2}$ $\left(-\frac{3}{2},0\right)$

5.3) $\frac{14}{3}=\frac{2}{x+1}+4$ $(x=k)$

$\frac{14}{3}-4=\frac{2}{x+1}$

$\frac{14}{3}-\frac{12}{3}=\frac{2}{x+1}$

$\frac{2}{3}=\frac{2}{x+1}$

$x=2 \Longrightarrow$ $\boxed{k=2}$

5.4) The y part of C = the horizontal asymptote

$\therefore C(2,4)$

5.5) $g(x)=a(x+p)^2+q$

$g(x)=a(x-2)^2+4$, substitute $(0,0)$

$g(x) = a(x-2)^2 + 4$, substitute (0,0)

$0 = a(0-2)^2 + 4$

$0 = 4a + 4$

$-4 = 4a$

$a = -1$

5. b) $x \in \left(-\infty, -\frac{3}{2}\right] \cup (-1, 0) \cup (4, \infty)$